The 'diffusion' of light and angular distribution in the laser equipped with a multilobe mirror

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Abstract

The distribution of radiation is investigated for the modeless laser having a multilobe mirror with the lobes (planes) inclined by small angles to optical axis. It is shown that change of the direction resulting from many passages of a ray through the optical system including a multilobe mirror may be described as Brownian walk of a point along the plane or equivalently as a solution of the two-dimensional diffusion equation. Boundary conditions for the diffusion equation may be approximately formulated as null conditions at some angle which, if being reached during the walk, guarantees that the ray escape from the optical system. In the framework of this approximation an explicit formula for the distribution of the outgoing ray in different angles is derived. After many passages through the optical system the angular distribution tends to some universal function. In the case of the round mirror it may be presented by the Bessel function of order zero.

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1 Introduction

It is known [1] that the angular distribution of the radiation of the laser with two plane-parallel mirrors is nonregular (inhomogeneous) because of nonideal mirrors and inhomogeneity of the active medium. This defect is absent in so called modeless lasers0 [2] having no longitudinal modes structure. Another configuration of modeless laser was proposed in the work [3]. In such a laser one of the plane mirrors is replaced by an multilobe mirror having the lobes (planes) slightly inclined to the optical axis (Fig.1 a). It was experimentally shown [3, 4] that this replacement makes the angular distribution of the laser radiation smooth (homogeneous). There is no longitudinal modes in this laser too. Light is mixed and scattered in the cavity of a multilobe mirror laser.

The form of the angular distribution in the multilobe laser was analyzed in the work [5] by consideration of geometry of the rays crossing the optical system with the multilobe mirror many times. Such an analysis proves to be complicated, may be carried out only for some simplest configurations of the multilobe mirror and gives no explicit formula for the angular distribution. In the present paper we shall analyze the case of an arbitrary multilobe mirror and calculate an approximate angular distribution in an explicit analytical form. The idea of the method is following.

In each double passage through the optical system the ray is deflected by a definite angle in one of a number directions depending on the inclination of the mirror the ray was reflected by. Change of the direction of the ray during multiple passage through the system have therefore the character of random walk and may be described mathematically by two-dimensional diffusion equation. Finiteness of the aperture of the optical system may be approximately taken into account by null boundary conditions at the boundary of some finite region of the plane. The region is a disk if the optical system has axial symmetry. In this case the problem may be easily solved by the method of separation of variables so that and explicit expression in the form of an infinite sum may be obtained for the resulting angular distribution.

Moreover, since each term of the sum exponentially decays with the number of passages increasing and the exponents are different, only one of the terms (having the minimal exponent) dominates in the case of a large number of passages. Thus, asymptotic (corresponding to large number of passages) angular distribution of the radiation turns out to be universal and is described by the Bessel function of order zero.

The reflection of the ray by the mirrors is considered in the framework of geometric optics and therefore the method is applicable only in the case of large Fresnel number, $a^2/\lambda L \gg 1$ (here λ is the maximal wave length in the wave packet, a the radius of the mirror and L the distance between the multilobe mirror and the opposite plane mirror closing the optical system). This means that the ratio λ/a should be much smaller than the ratio a/L. We assume too that the thickness of the multilobe mirror $h \ll L$.

2 Double passage of a ray through the optical system

If a ray of light which is directed along the unit vector $\mathbf{k}$ falls onto the plane mirror having the unit normal vector $\mathbf{n}$, then the reflected ray is directed along the unit vector (Fig. 1 b)

$$\mathbf{k}_1 = \mathbf{k} - 2(\mathbf{k}\mathbf{n})\mathbf{n} \tag{1}$$

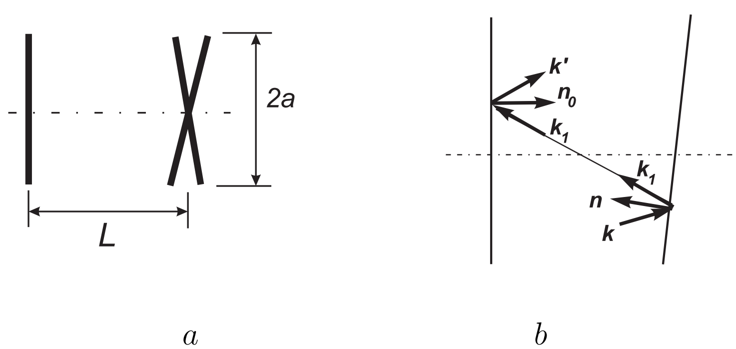

Figure 1: An optical system equipped by a multilobe mirror: a) The general scheme; b) the form of the ray reflected by a lobe of the multilobe mirror and then by the closing mirror.

Let $\mathbf{n}_0$ be the unit vector along the axis of the optical system. Assume that the vector $\mathbf{k}$ characterizing the ray is close to $\mathbf{n}_0$ and the normal $\mathbf{n}$ to the mirror is close to $-\mathbf{n}_0$. Then we can represent these vectors in the form

$$\mathbf{k} = \mathbf{n}_0 + \vec{\alpha}, \quad \mathbf{n} = -\mathbf{n}_0 + \vec{\gamma} \tag{2}$$

where the vectors $\vec{\alpha}$ and $\vec{\gamma}$ are orthogonal to $\mathbf{n}_0$ and the terms of the second order in $\vec{\alpha}$ and $\vec{\gamma}$ are omitted. The absolute values of the vectors $\vec{\alpha}$ and $\vec{\gamma}$ are equal to the angles of inclination of the ray and the mirror normal to the optical axis. The directions of these vectors show in which direction the ray and the mirror are inclined. Call therefore the vectors $\vec{\alpha}$ and $\vec{\gamma}$ 'angle vectors' or simply 'angles'.

We shall assume that the inclination angles are small so that only first order terms may be conserved in calculations. In this approximation $\mathbf{kn} = -1$ and therefore the reflection law (1) takes the form

$$\mathbf{k}_1 = -\mathbf{n}_0 + \vec{\alpha} + 2\vec{\gamma}. \tag{3}$$

Let now the reflected ray (having the direction $\mathbf{k}_1$) be again reflected by the mirror closing the optical system from the opposite side. If this mirror is orthogonal to the optical axis, i.e. it has normal $\mathbf{n}_0$, then the direction of the ray after this second reflection is characterized by the vector

$$\mathbf{k}' = \mathbf{k}_1 - 2(\mathbf{k}_1\mathbf{n}_0)\mathbf{n}_0 = \mathbf{n}_0 + \vec{\alpha} + 2\vec{\gamma}. \tag{4}$$

Finally we see that the ray characterized by the vector angle $\vec{\alpha}$ converts after double passage through the optical system to the ray corresponding to the new vector angle $\vec{\alpha}'$:

$$\vec{\alpha} \to \vec{\alpha}' = \vec{\alpha} + 2\vec{\gamma}. \tag{5}$$

Here the vector angle $\vec{\gamma}$ characterizes inclination of the mirror which reflected the ray. This mirror is only one lobe of the multilobe mirror. In each double passage the ray is reflected from one of the lobes chosen randomly.

3 'Diffusion' of the inclination angle and the angular distribution of rays.

Assume that the multilobe mirror contains s mirrors (lobes) characterized by the vector angles $\{\vec{\gamma}_1, \vec{\gamma}_2, \ldots, \vec{\gamma}_s\}$. After each double passage the ray may be reflected by one of these mirrors. If the mirror with number i, $i = 1, 2, \ldots, s$ is chosen, the direction of the ray is changed as follows:

$$\vec{\alpha} \rightarrow \vec{\alpha}' = \vec{\alpha} + 2\vec{\gamma}_i, \tag{6}$$

In each double passage the number i is chosen randomly according some probability distribution. In the simplest case the probabilities of all lobes are equal, $p_i = 1/s$. We shall consider only this case.

After N double passages an initial inclination angle changes as

$$\vec{\alpha} \rightarrow \vec{\alpha}' = \vec{\alpha} + \vec{\eta} \tag{7}$$

where

$$\vec{\eta} = \vec{\xi}_1 + \vec{\xi}_2 + \ldots + \vec{\xi}_N$$

is a random variable equal to the sum of N random variables $\vec{\xi}_i$. All variables $\vec{\xi}_i$ have equal distributions. Each of them takes the values $\{2\vec{\gamma}_1, 2\vec{\gamma}_2, \ldots, 2\vec{\gamma}_s\}$ with corresponding probabilities. If the probabilities of all values are equal to each other, $p_i = 1/s$, then the expectation values, variances and covariations for the components of each of the (vector) random variables $\vec{\xi}_i$ are

$$M\xi_x = \frac{1}{s}\sum_i 2\gamma_{ix}, \qquad M\xi_y = \frac{1}{s}\sum_i 2\gamma_{iy},$$

$$D\xi_x = \frac{1}{s}\sum_i (2\gamma_{ix} - M\xi_x)^2, \qquad D\xi_y = \frac{1}{s}\sum_i (2\gamma_{iy} - M\xi_y)^2,$$

$$\mathrm{cov}(\xi_x, \xi_y) = \frac{1}{s}\sum_i (2\gamma_{ix} - M\xi_x)(2\gamma_{iy} - M\xi_y).$$

We shall assume for the aim of simplicity that the set of vectors $\{\vec{\gamma}_i\}$ is symmetrical so that

$$M\xi_x = M\xi_y = 0, \quad D\xi_x = D\xi_y = \sigma^2, \quad \mathrm{cov}(\xi_x, \xi_y) = 0.$$

Then the x- and y- components of the random variables ξ_i have the same characteristics and are independent from each other. For the random variable $\vec{\eta}$ (which is the sum of $\vec{\xi}_i$) we have

$$M\vec{\eta} = 0, \quad D\eta_x = D\eta_y = N\,\sigma^2 \quad \mathrm{cov}(\eta_x, \eta_y) = 0.$$

Change of the angle described as in (7) is nothing else than Brownian walk of a point in a plane. If N is large, then, owing to the central limiting theorem of the probability theory, the distribution in various values $\mathbf{r} = (x, y)$ of the random variable $\vec{\eta}$ may be approximated by the normal distribution:

$$p_N(x, y)dxdy = \frac{1}{2\pi N\sigma^2} \exp\left(-\frac{x^2 + y^2}{2N\sigma^2}\right) dxdy. \tag{8}$$

The probability distribution for the components of the vector angle $\vec{\alpha}'$ (for the given initial angle $\vec{\alpha}$) is found readily. If we denote these components by the same letters x, y, then this probability has the form

$$P_N(x, y|\vec{\alpha})dxdy = \frac{1}{2\pi N\sigma^2} \exp\left(-\frac{(x - \alpha_x)^2 + (y - \alpha_y)^2}{2N\sigma^2}\right) dxdy.$$

If an arbitrary initial distribution of the angles $\vec{\alpha}$ is given, then the final distribution is

$$P_N(x, y) = \int P_N(x, y|\vec{\alpha})\, P(\vec{\alpha})d\alpha_x d\alpha_y.$$

If $N\sigma^2 = t$ is considered as time, then the distribution function $P(t, x, y) = P_N(x, y)$ satisfies the diffusion equation:

$$\frac{\partial P}{\partial t} = \frac{1}{2}\left(\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2}\right). \tag{9}$$

In the preceding consideration is has been assumed that the angle $\vec{\alpha}'$ may become, as a result of the random process, arbitrarily large. In reality it is restricted by finiteness of the aperture. This may be taken into account by the corresponding boundary conditions for the equation (9).

4 Angular distribution for finite aperture

When after a number of double passages the inclination angle becomes sufficiently large, the ray will be lost owing to the finiteness of aperture. In the language of Brownian walk this means that the point disappears and takes no part in further random process. If we have initially many Brownian particles, part of them is lost because of finite aperture so that the final number is less. In terms of the probability distribution, its norm decreases with time.

At first glance, this argument leads to null conditions at the boundary of some finite region in the plane of Brownian walk. Adding this boundary condition to the equation (9), we could find the distribution taking into account finiteness of the aperture. In the case of axial symmetry of the optical system the permitted region might be a disc of the radius restricting the maximum absolute value of the angle depending on the aperture. One is attempted to impose null conditions at the angle equal to $\Gamma = a/L$ where a is the radius of the mirror and L distance between the opposite mirrors of the optical system.

This is however not so simple. Precisely speaking, no null boundary condition may be formulated for the angle $\vec{\alpha}'$. Yet we shall show that this may be made approximately. The null initial condition must be imposed at some critical angle Γ_c depending on Γ and the angle σ characterizing deflection at each double passage.

The precise formulation of the problem is possible only in terms of the joint distribution $P(\alpha, \vec{\rho})$ in the angle $\vec{\alpha}$ of inclination of the ray and position $\vec{\rho}$ of the point of reflection this ray at the mirror. It must be required that this distribution function be zero when $\vec{\rho}$ takes values corresponding to the edge of the mirror. The problem is that the equation for the function $P(\alpha, \vec{\rho})$ is complicated and cannot be dealt efficiently.

Instead of this correct consideration we shall restrict ourselves by the function $P(\alpha)$ and the diffusion equation for it. As for the boundary conditions, we shall find, up to the order of magnitude, the angle Γ_c such that when this angle of inclination is reached, the ray is with great probability lost in some time after this.

To find the critical angle Γ_c, we shall argue in the following way. Let the inclination angle Γ_0 is reached in the course of Brownian walk. If this angle leaves unchanged during next passages through the system, then after each double passage the point of the reflection at the mirror is replaced by $2L\Gamma_0$. Then after $N_0 = \Gamma/\Gamma_0$ double passages the point of reflection is replaced by the value $2LN_0\Gamma_0 = 2L\Gamma = 2a$ so that the ray will certainly escape from the optical system.

However after reaching the value Γ_0 the inclination angle will not stay constant but change in the course of Brownian walk. The preceding conclusion that the ray will be lost in N_0 double passages will be valid if the change of the angle is much less than Γ_0. Making use of typical Brownian replacement $\sqrt{N_0}\sigma$, we have inequality $\sqrt{N_0}\sigma \ll \Gamma_0$ as the condition that the ray will be lost. Equivalently this condition may be written as $\Gamma_0 \gg \Gamma_c$ where

$$\Gamma_c = (\Gamma\sigma^2)^{1/3}$$

Vice versa, if $\Gamma_0 \ll \Gamma_c$ then the random changes of the angle Γ_0 in the course of Brownian walk will radically change it before the ray will be lost. In this case the fact that the angle reached the value Γ_0 has no special significance.

We see that the angle Γ_c is critical in the sense that reaching this angle in the course of Brownian walk leads with great probability to loss of the ray. This means that the distribution function $P(\alpha)$ must be small for $|\alpha| = \Gamma_c$. Approximately we may take it to be zero for such arguments. This means imposing null boundary conditions at the boundary of the disc of radius Γ_c.

Thus, the diffusion equation (9) should be solved with the null boundary conditions at the boundary of the disc of radius Γ_c:

$$P(t,x,y) = 0 \quad \text{when} \quad x^2 + y^2 = \Gamma_c^2.$$

This solution is easily obtained with the help of the separation of variables in the polar coordinates. It has the form

$$\begin{aligned} P(t,r,\varphi) &= P(t,x,y) \\ &= \sum_{n=0}\sum_{m=1} e^{-\omega_{mn}^2\, t}\, J_n(k_{mn}r)\, \alpha_{mn} \cos n\varphi \\ &+ \sum_{n=1}\sum_{m=1} e^{-\omega_{mn}^2\, t}\, J_n(k_{mn}r)\, \beta_{mn} \sin n\varphi \end{aligned} \qquad (10)$$

where the following notations are used:

$\omega_{mn}^2 = \dfrac{1}{2}\, k_{mn}^2, \quad k_{mn} = \mu_{mn}/\Gamma_c, \quad \mu_{mn}$ are the roots of the Bessel function $J_n(r)$.

The coefficients α_{mn}, β_{mn} are determined by initial conditions as follows:

$$P(0,r,\varphi) = \sum_{n=0}\sum_{m=1} (\alpha_{mn} \cos n\varphi + \beta_{mn} \sin n\varphi) J_n(k_{mn}r).$$

Asymptotically (at large $t = N\sigma^2$) the first term in the formula (10) dominates because it corresponds to the minimal root of the Bessel functions and therefore minimal exponent. Therefore after a large number of double passages

$$p(t,r,\varphi) = P(t,x,y) = P_N(x,y) \approx \alpha_{10} e^{-\omega_{10}^2\, t}\, J_0(\mu_{10} r/\Gamma_c). \qquad (11)$$

It is only numerical factor α_{10} which depends on initial conditions in this expression. Therefore, asymptotic (for large N) angular distribution of the radiation issued by the optical system with the multilobe mirror does not depend on initial conditions and is described by the Bessel function of zero order.[1]

5 Conclusion

In the present paper we have considered (in the framework of the geometric optics) the rays of light in an optical system having an usual plane mirror from one side and a multilobe mirror from another side. We evaluated the probability that the outcoming ray has a definite inclination angle. It was shown that the reflection of a ray from one of many lobes of the multilobe mirror inclines it in one of many directions. As a result, the inclination angle undergoes Brownian walk. The resulting probability distribution satisfies then the diffusion equation, with the number of passages playing the role of time.

If there are many input rays, then the same function which describes the probability distribution gives the angular distribution of the outcoming rays. It was shown that the distribution alters when the number of passages of the ray through the optical system increases. Asymptotically for very large number of passages the distribution tends to some universal distribution which possesses the same symmetry as the lobes of the multilobe mirror (axial symmetry in the considered case).

If a (modeless) laser is constructed on the basis of such an optical system, its radiation will have distribution identical with the asymptotic distribution we have found. The angular distribution of the multilobe mirror laser proves to be stable under perturbations, therefore the distribution (11) maintains in the non-ideal conditions (for example with inhomogeneous active medium). This explains why the multilobe mirror laser have smooth (homogeneous) angular distribution [3, 4].

Several methods could be applied to increase the homogeneity of active multilobe mirror lasers. Among them 1) the increasing of the Fresnel number

[1] In the very special case when initial conditions correspond precisely zero coefficient α_{10}, the asymptotic distribution is determined by the next term of the sum corresponding to the larger root of the Bessel function.

(i. e. the increasing of the laser aperture and the decreasing of the cavity length and wavelength), 2) the increasing of the number of lobes (planes) of the multilobe mirror, 3) the application of the active medium with sufficient broad bandwidth (for example Neodymium glass active elements). The above developed method of calculations may be generalized to include these modifications increasing the homogeneity of a laser beam.

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